



香港中文大學  
The Chinese University of Hong Kong

Institute of Theoretical Computer Science and Communications

*CSE- ITCSC-INC Seminar*

**Graph Connectivity, Network Coding, Expander Graph,  
and Matrix Rank**

By

**Prof. Lap Chi Lau,**

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*April 12, 2012, Thursday*

*2:30 p.m. - 3:30 p.m.*

*Rm121, 1/F., Ho Sin Han Engineering Building, CUHK*

**Abstract:** We design fast algorithms for computing graph connectivity and computing matrix rank by using ideas from network coding and constructions from expander graphs.

First we present a new algebraic formulation to compute edge connectivities in a directed graph, using the ideas developed in network coding. This reduces the problem of computing edge connectivities to solving systems of linear equations, thus allowing us to use tools in linear algebra to design new algorithms. Using the algebraic formulation we obtain faster algorithms for computing single source edge connectivities and all pairs edge connectivities, in some settings the amortized time to compute the edge connectivity for one pair is sublinear. Through this connection, we have also found an interesting use of expanders and superconcentrators to design fast algorithms for graph connectivity problems.

Then we show how to use the above ideas to design fast algorithms for computing matrix rank, a basic problem in numerical linear algebra. Given a  $m \times n$  matrix  $A$ , one can find  $k$  linearly independent columns in  $O(mnk^{\omega-2})$  time by Gaussian elimination, where  $\omega < 2.38$  is the matrix multiplication exponent. We present an  $O(|A| + k^{\omega})$  time randomized algorithm to find  $k$  linearly independent columns, where  $|A|$  is the number of nonzero entries of  $A$ . This is considerably faster when  $k$  is small, for example when the matrix  $A$  is rectangular it is always faster than Gaussian elimination. We mention various algorithmic applications in numerical linear algebra and combinatorial optimization.

Joint work with Ho Yee Cheung, Tsz Chiu Kwok, Kai Man Leung.

**Biography:** Lap Chi Lau is an assistant professor of the Department of Computer Science and Engineering in The Chinese University of Hong Kong. He was born in Hong Kong and received his B.Sc. degree in computer science from CUHK. He received his M.Sc. and Ph.D. degrees in computer science from the University of Toronto. His research interest is in algorithm design and combinatorial optimization.

\*\*\*\*\* ALL ARE WELCOME \*\*\*\*\*

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